CDCL(Crypto) SAT Solvers for Cryptanalysis

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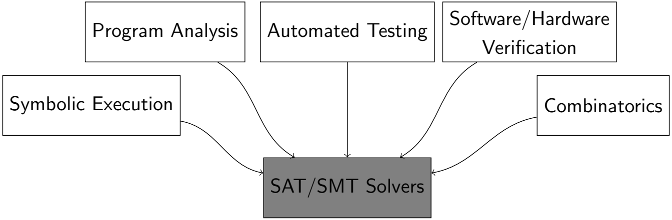
University of Waterloo

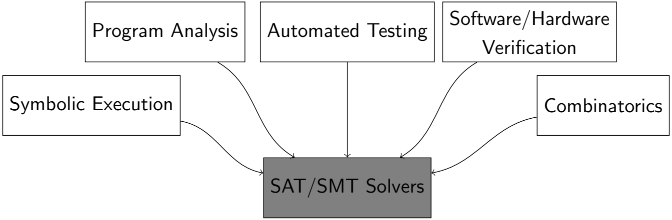
Motivation

- **SAT Solvers**: Powerful general purpose search tools



Motivation

- **SAT Solvers**: Powerful general purpose search tools
- **Cryptanalysis**: Searching a huge search space for a secret key/value



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Question

Can we tailor internals of a SAT solver for a specific cryptographic problem to improve the solving time?

Outline

Boolean SAT Solvers

- CDCL SAT Solvers

The CDCL(Crypto) Framework

- Programmatic SAT Architecture

Case Studies

- Algebraic Fault Attack
- Differential Cryptanalysis

Boolean SAT Solvers

Boolean SATisfiability

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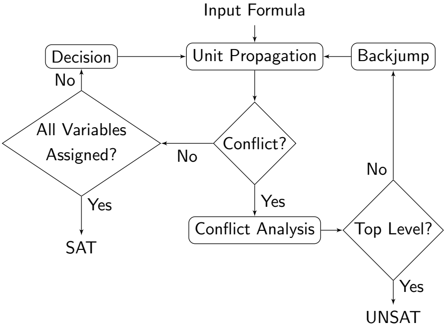
- A formula is satisfiable, if there exists an assignment to the variables which the formula true.
- Example: $(x \vee y \vee \neg z) \wedge (\neg x \vee \neg y) \wedge z$

CDCL SAT Solver

Unit propagation



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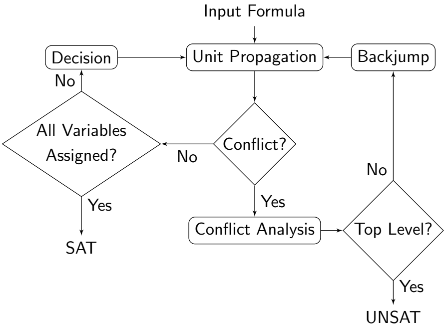

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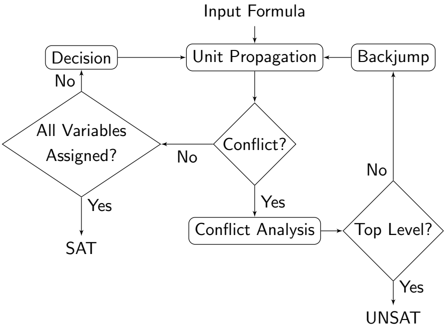

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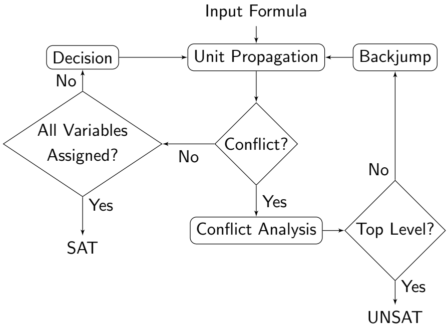

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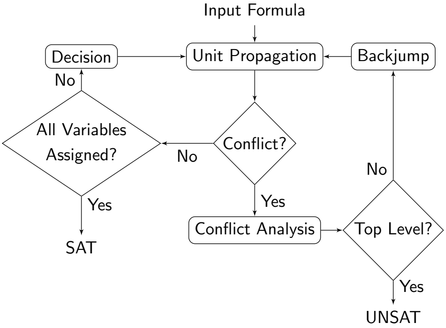
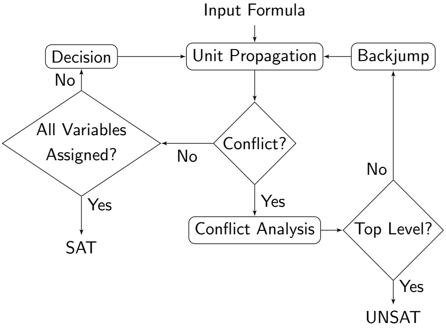
- $\neg z, \quad \neg z \rightarrow x, \quad x \rightarrow \neg y$



CDCL SAT Solver

Decision/Branching heuristics

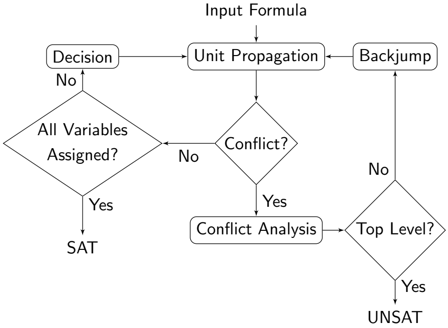
- Pick an unassigned variable and set it to False/True



CDCL SAT Solver

Conflict analysis

- Find the root cause of conflict
- Encode it as a clause and add it back to the formula



The CDCL(Crypto) Framework

Lost in Translation

- When encoding a constraint into SAT, some higher level properties might be lost
- Example: consider a pseudo-Boolean constraint $C : x + y \leq 0$
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- "Better" encoding vs. "Better" Propagation

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- Propagation callback
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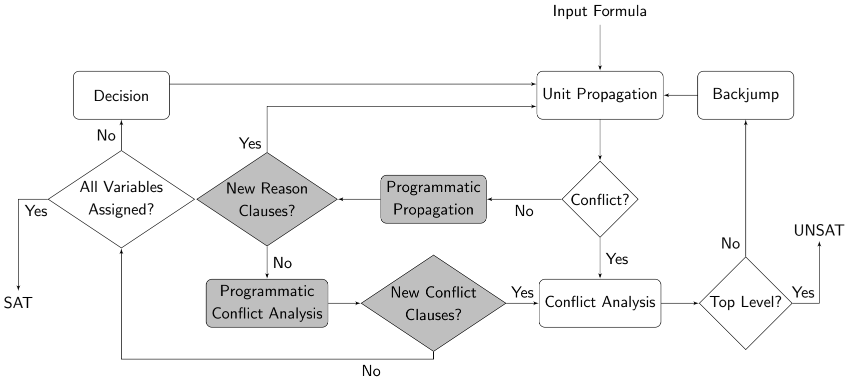
  - Checks for implied literals that are missed by unit propagation
- Conflict analysis callback
  - Called after *propagation* is done
  - Checks if partial assignment cannot be extended to a full solution

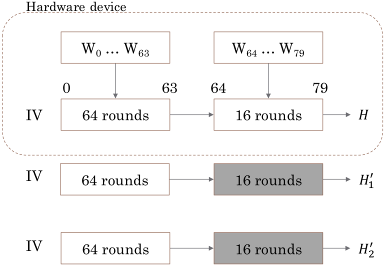
Programmatic SAT



Case Studies

Hardware Fault Injection

- SHA-1 hash function



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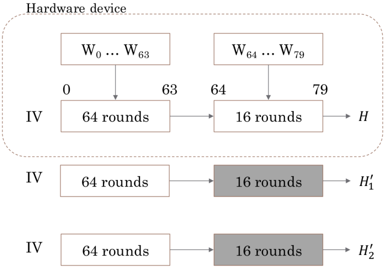
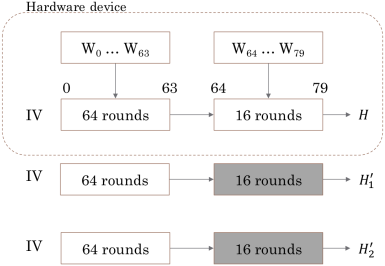


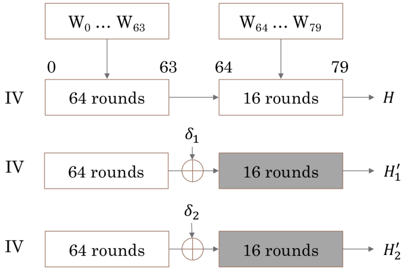
Hardware Fault Injection

- SHA-1 hash function
- Induce a fault in a target register
- Using heat, EM, laser, ...



Algebraic Fault Analysis

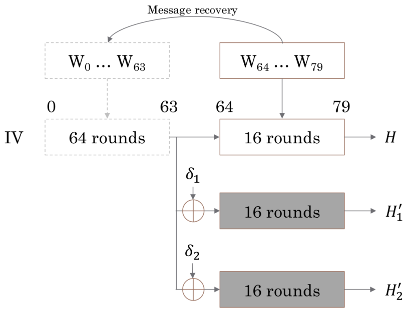
- $H = f_{0..79}(IV, W_{0..79})$
- This equation system will be encoded into CNF.
- Fault model: Constraints on δ_i.



$$H'_i = f_{64..79}(f_{0..63}(IV, W_{0..63}) \oplus \delta_i, W_{64..79})$$

Algebraic Fault Analysis

- Abstract away the common parts
- Verification of the solution will be needed

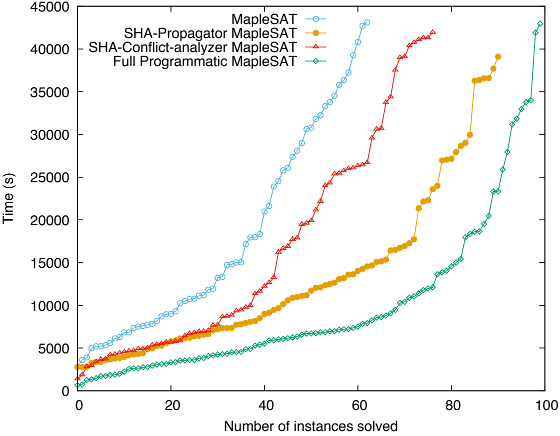


Algebraic Fault Analysis - Programmatic Approach

- Base SAT solver: MapleSAT
- Programmatic conflict analyzer
  - Embedding the verification loop
  - As soon as message word variables are set, they are ready to be verified
  - Early embedded check vs. Straightforward check after solving completely
- Programmatic propagator
  - Improving the propagation flow of multi-operand additions
  - Generating *reason clauses* in each column addition when output bits are missed

Algebraic Fault Analysis - Results

- Recovering SHA-256 message bits
- 14.3x speed-up on average
- 17 fewer faults were needed compared to the previous works



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- Analyzing how a difference at the input propagates to a difference at the output.
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- Differential Path: A trace of differentials over smaller steps in the function
- Collision: a differential path with final difference equal to zero.

- Guess-and-determine solvers:
    - Very similar search approach to SAT solvers
    - Dedicated propagators for differential propagation rules
    - Dedicated branching heuristics
    - State-of-the-art results on SHA-256 collision (31 steps)

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- Example rule: $r = IF(x, y, z)$, `-` $\leftarrow$ `---`, `x` $\leftarrow$ `-xx`.
- Full representation yields a blow up in size of the encoding
- An opportunity for Programmatic propagation

Differential Cryptanalysis - Results

- Collision on round-reduced SHA-256
- Modified the starting differential path of [Pro16]
- Base solver: MapleSAT
- Programmatic Propagator: Implemented a subset of differential propagation rules
- Programmatic conflict analyzer: Detects impossible differentials

- Found collision for 25 rounds of SHA-256 using MapleSAT(Crypto) in $\sim$3.5 hours

Conclusions

- A framework on top of CDCL SAT solvers to implement cryptographic reasonings.
- Showcased the power of the framework in two cryptanalysis tasks.
- A bridge between two ends of a spectrum:
    - Performance of dedicated cryptanalysis tools
    - Flexibility and search power of SAT solvers
- Beating state-of-the-art in some cases but still a long way to match state-of-the-art in other cases

Thanks!

Questions?

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- A *theory solver* checks if the assignment is a solution to the original formula (and if not, why not).
- Here the T-solver can return the clause $\neg A$ (i.e. $x^2 \geq 0$).